NORMANHURST BOYS' HIGH SCHOOL
N E W S O U T H W A L E S

Student Number

Class: 12M 1 2 3 4 5 (Please Circle)

2013
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Begin each question on a separate writing booklet

Total marks - 70

Section I Pages 2-5

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 6-11

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Students are advised that this is a school-based examination only and cannot in any way guarantee the content or format of future Higher School Certificate Examinations.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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(1) The height of a giraffe has been modelled using the equation:
$H = 5.40 - 4.80e^{-kt}$
where H is the height in metres, t is the age in years and k is a positive constant.
If a 6 years old giraffe has a height of 5.16 metres, find the value of k, correct to 2 significant figures.

(A) 0.05

(B) 0.24

(C) 0.50

(D) 4.8

(2) What is the value of $\lim_{x \to 0} \left(\frac{sin\frac{1}{3}x}{2x} \right)$

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 6

(3) Which of the following equates to the expression $\frac{1-e^{3x}}{1-e^{2x}}$.

(A) $1+\frac{e^{2x}}{1+e^{x}}$

(B) $1-e^{x}$

(C) $1+e^{x}+e^{2x}$

(D) None of the above

(4) The point P divides the interval $A(\frac{17}{3}, 2)$ to $B(-3, 4)$ **externally** in the ratio $2:3$. Which one of the following is the coordinates of point P?

(A) $(-23, 2)$

(B) $(-9, -12)$

(C) $(9, 0)$

(D) $(23, -2)$

(5) A curve is defined by the parametric equations $x = \sin 2t$ and $y = \cos 2t$. Which of the following, in terms of t, equates to $\frac{dy}{dx}$?

(A) $-\tan 2t$

(B) $2\tan 2t$

(C) $2\sin 4t$

(D) $\cos 4t$

(6) Which of the following is the inverse function of $y = \frac{x-4}{x-2}$, $x \neq 2$?

(A) $y = \frac{x-2}{x-4}$

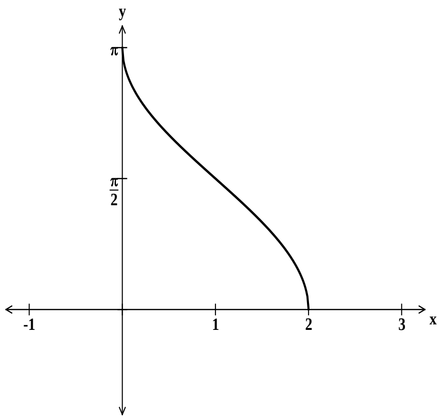
(B) $y = f^{-1}(y)$

(C) $y = \frac{2(x-2)}{x-1}$

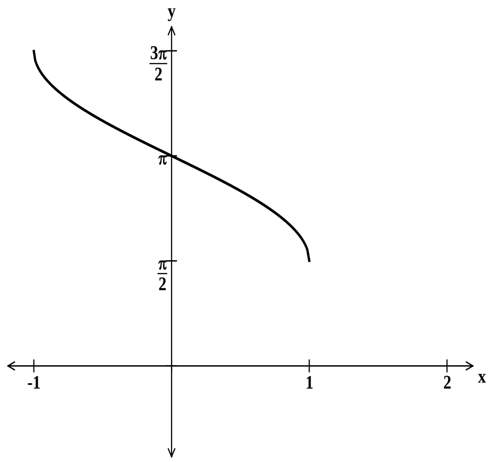
(D) $y = \frac{x+4}{x+2}$

(7) Which of the following represents the graph of $y = \cos^{-1}(x+1)$.

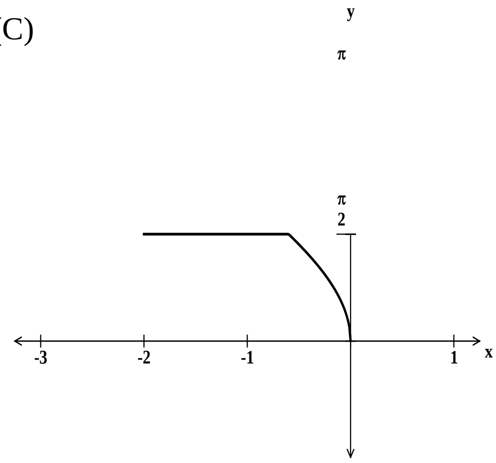
(A)



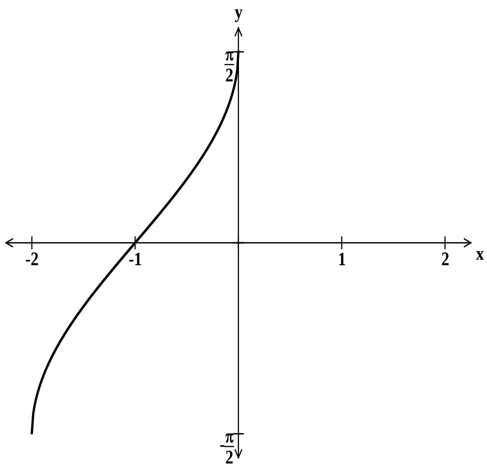
(B)



(C)



(D)



(8) Given that $xy = x + 1$, the definite integral $\int_3^5 x\ dy$ equates to:

(A) e^2

(B) $\ln 2$

(C) $-\ln\left(\frac{5}{3}\right)$

(D) $e^{\frac{5}{3}}$

(9) The motion of a particle moving along the $x-axis$ executes simple harmonic motion. The maximum velocity of the particle is $4\ m/s$ and the period of motion is π seconds. Which of the following could be the displacement equation for this particle?

(A) $x = 4\cos \pi t$

(B) $x = -\sin 2t$

(C) $x = 2\cos 2t$

(D) $x = 2 + \cos 2t$

(10) A particle moves with a velocity $v\ m/s$ where $v = \sqrt{x^2 + 1}$. Given that $x > 0$, which of the following is equal to the acceleration of the particle when $v = 4\,m/s$.

(A) $\sqrt{17}\,m/s^2$

(B) $-3\,m/s^2$

(C) $\sqrt{15}\,m/s^2$

(D) $2\sqrt{17}\,m/s^2$

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Find the exact value of $\sin 75^o$. **2**

(b) Given that the acute angle between the lines $y = mx$ and $2x - 3y = 0$ is 45^o, find possible value(s) of *m*. **3**

(c) Using the substitution $u = 1 + x^2$, or otherwise, evaluate **3**

$$\int_0^{\sqrt{8}} \left(\frac{x}{\sqrt{1+x^2}} \right) dx .$$

(d) Solve the following inequality for *x*: **3**

$$\frac{1}{x} + \frac{x}{(x-2)} < 0$$

(e) (i) On the same number plane, graph the following functions: **2**

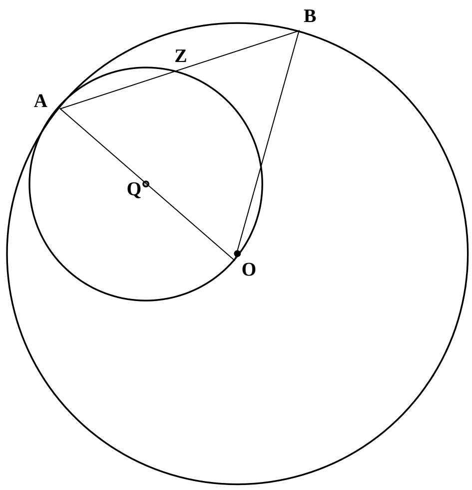
$y = 4 - x^2$ and $y = |3x|$

(ii) Hence or otherwise solve $4 - x^2 \le |3x|$ **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find the exact value of $\sin\left(2\cos^{-1}\dfrac{\sqrt{3}}{4}\right)$. **2**

(b) AB is a chord of a circle centre O. AO is a diameter of a circle centre Q. Z is the point where the circle centre Q meets AB.



NOT TO SCALE

(i) Explain why $AO = OB$. **1**

(ii) Hence or otherwise, prove that $AZ = ZB$. **2**

(c) The quadratic equation $x^2 - 4x + 9 = 0$ has roots $\tan A$ and $\tan B$. Hence, find the size(s) of $\angle(A+B)$, noting that $0 \le A + B \le 360^o$ (*leave your answer to the nearest degree*). **3**

(d) (i) By use of long division, find the remainder, in terms of a and b when $P(x) = x^4 + 3x^3 + 6x^2 + ax + b$ is divided by $x^2 + 2x + 1$. **2**

(ii) If this remainder is $3x + 2$, find the values of a and b. **1**

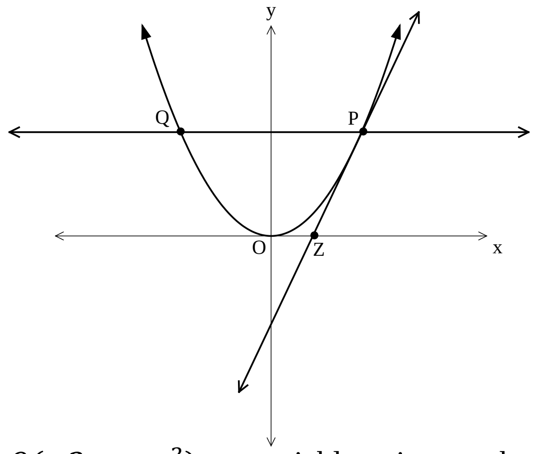
(e) (i) Prove that $\sin A \cos A \cos 2A = \dfrac{1}{4}\sin 4A$. **2**

(ii) Hence or otherwise solve $\sin A \cos A \cos 2A = 0$, for $0 \le A \le \frac{\pi}{2}$. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Prove by mathematical induction that $5^n \geq 1 + 4n$ for all integers $n \geq 1$. **3**

(b)



$P(2ap, ap^2)$ *and* $Q(-2ap, ap^2)$ are variable points on the parabola $x^2 = 4ay$. The line PQ is parallel to the $x-axis$. The tangent at P meets the $x-axis$ at Z.

(i) Show that the equation of the tangent at P is given by $y = px - ap^2$ **2**

(ii) Hence show that $Z = (ap, 0)$. **1**

(iii) Find the locus of midpoints of QZ. **2**

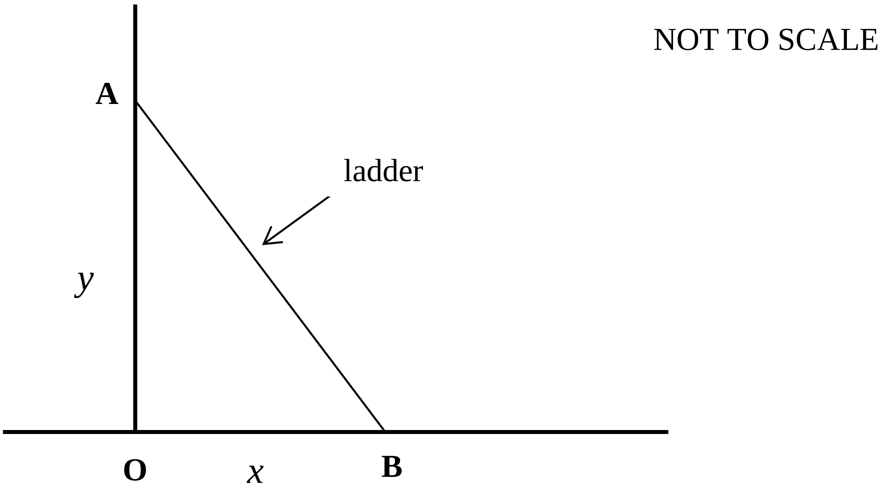
(c) (i) Graph the function $y = 2\tan^{-1}(x)$. **1**

(ii) Graphically show why $2\tan^{-1}(x) - \frac{x}{4} = 0$ has one root, for $x > 0$. **1**

(iii) Taking $x_1 = 10$ as a first approximation to this root, use one application of Newton's method to find a better approximation, correct to 2 decimal places. **2**

Question 13 continues on page 9

13 (d) **3**



A ladder AB, 5 metres long, is leaning against a vertical wall OA, with its foot B, on horizontal ground OB. The distances OB and OA are x and y metres respectively. x and y are related by the equation $x^2 + y^2 = 25$.

The foot of the ladder begins to slide along the ground away from the wall at a constant speed of 1 metre per second.

Find the speed at which the top of the ladder A is moving down the wall at the time when the top of the ladder is 4 metres above the ground.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is travelling in a straight line. Its displacement $(x\,cm)$ from O at a given time $(t\sec)$ after the start of motion is given by: $x = 2 + \sin^2 t$.

(i) Prove that the particle is undergoing simple harmonic motion. **2**

(ii) Find the centre of motion. **1**

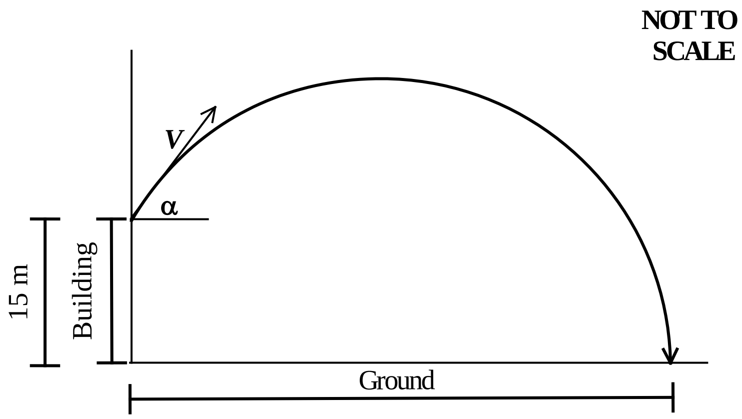
(iii) Find the total distance travelled by the particle in the first $\dfrac{3\pi}{2}$ seconds. **2**

(b) A shade sail with corners *A*, *B* and *C* is shown in diagram 1, supported by three vertical posts. The posts at corners *A* and *C* are the same height, and the post at corner *B* is 2.4 m taller. Diagram 2 shows the sail in more detail. *D* is the point on the taller post horizontally level with the tops of the other two posts. **3**
AD = 6.4 m and *DC* = 5.2 m. $\angle ADC = 125^\circ$.
Find the area of the shade sail *ABC* (*leave your answer to 1 decimal place)*

Question 14 continues on page 11

Question 14 (continued)

(c) Over 80 years ago, during training exercises, the Army fired an experimental missile from the top of a building 15 m high with initial velocity (v) where $v = 130 m/s$, at an angle (α) to the horizontal. Noting that $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ and taking $g = 10 m/s^2$



Assume that the equations of motion of the missile are
$\ddot{x} = 0$ and $\ddot{y} = -10$

(i) Show that $\dot{x} = 120$ and $\dot{y} = -10t + 50$. **2**
Hence write down the equations of x and y.

(ii) The rocket hit its intended target when its velocity reached $60\sqrt{5}$ m/s. **2**
Find the horizontal distance that the missile travelled to hit its target.

(iii) The rocket was designed to hit its target once the angle to the horizontal of its flight path in a downward direction lies between 20^o and 30^o. Find the range of times after firing that this could happen. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Normanhurst Boys High School

2013 HSC TRIAL EXAMINATION

MATHEMATICS EXTENSION 1 – MARKING GUIDELINES

Section I

Question	Marks	Answer	Outcomes Assessed
1	1	~~B~~C	O1
2	1	A	O1
3	1	A	O1
4	1	D	O3
5	1	A	O4
6	1	C	O4
7	1	C	O4
8	1	B	O5
9	1	C	O5
10	1	C	O5

Question 11 (15 marks)

11(a) (2 marks)
Outcomes Assessed: O3

Criteria	Marks
• Use sine of the sum	1
• Correct answer	1

Answer

$$\sin 75^\circ = \sin(45^\circ + 30^\circ)$$
$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

11(b) (3 marks)
Outcomes Assessed: O3

Criteria	Marks
• Obtains correct gradients	1
• Correct substitution into formulae	1
• Correct answer	1

Answer

$$\tan 45^\circ = \left|\frac{m - \frac{2}{3}}{1 + \frac{2m}{3}}\right|$$
$$\left|\frac{3m-2}{3} \div \frac{3+2m}{3}\right| = 1$$
$$\left|\frac{3m-2}{3+2m}\right| = 1$$
$$\frac{3m-2}{3+2m} = 1 \quad or \quad \frac{3m-2}{3+2m} = -1$$
$$m = 5 \quad or \quad m = -\frac{1}{5}$$

11(c) (3 marks)
Outcomes Assessed: O5

Criteria	Marks
• Obtains correct limits	1
• Obtains $I = \frac{1}{2}\int_1^9 \left(\frac{du}{u^{\frac{1}{2}}}\right)$	1
• Correct answer	1

Answer

$$u = 1 + x^2$$
$$\frac{1}{2}du = x\,dx$$
$$x = \sqrt{8} \rightarrow u = 9$$
$$x = 0 \rightarrow u = 1$$

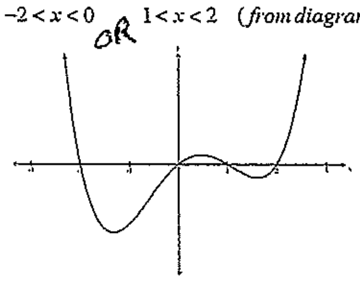
$$I = \int_0^{\sqrt{8}} \left(\frac{x}{\sqrt{1+x^2}}\right) dx$$
$$I = \frac{1}{2}\int_1^9 \left(\frac{du}{u^{\frac{1}{2}}}\right)$$
$$= \left[u^{\frac{1}{2}}\right]_1^9$$
$$= 3 - 1$$
$$= 2$$

11(d) (3 marks)
Outcomes Assessed: O2

Criteria	Marks
• Multiplies throughout by $x^2(x-2)^2$	1
• Obtain $x(x-2)(x+2)(x-1) < 0$	1
• Correct answer	1

Answer

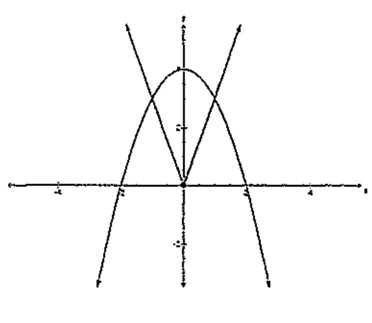
$$\frac{1}{x} + \frac{x}{(x-2)} < 0$$
$$x(x-2)^2 + x^3(x-2) < 0$$
$$x(x-2)\left[(x-2) + x^2\right] < 0$$
$$x(x-2)\left[x^2 + x - 2\right] < 0$$
$$x(x-2)(x+2)(x-1) < 0$$
$$-2 < x < 0 \quad \text{OR} \quad 1 < x < 2 \quad (\textit{from diagram})$$



11(e) (i) (2 marks)
Outcomes Assessed: O1

Criteria	Marks
• Correct graph for $y = 4 - x^2$	1
• Correct graph for $y = \lvert 3x \rvert$	1

Answer



11(e) (ii) (2 marks)
Outcomes Assessed: O1

Criteria	Mark
• Solve for both points of intersection	1
• Correct answer	1

Answer

Solve
$4-x^2=|3x|$
$4-x^2=3x \quad or \quad 4-x^2=-3x$
$x^2+3x-4=0 \quad or \quad x^2-3x-4=0$
$(x+4)(x-1)=0 \quad or \quad (x-4)(x+1)=0$
$x=1,-4 \qquad x=4,-1$
check solutions
correct solutions : $x=\pm 1$
hence from diagram $x<-1$ *or* $x>1$

Question 12 (15 marks)
12(a) (2 marks)
Outcomes Assessed: O4

Criteria	Marks
• Achieves $\cos\alpha=\frac{\sqrt{3}}{4}$ and $\sin\alpha=\frac{\sqrt{13}}{4}$	1
• Correct answer	1

Answer

Let $\alpha=\cos^{-1}\frac{\sqrt{3}}{4}$

$\therefore \sin\left(2\cos^{-1}\frac{\sqrt{3}}{4}\right)=\sin(2\alpha)$

Also $\cos\alpha=\frac{\sqrt{3}}{4}$, *hence* $\sin\alpha=\frac{\sqrt{13}}{4}$ *(pythagoras)*

$$\sin\left(2\cos^{-1}\frac{\sqrt{3}}{4}\right)=\sin(2\alpha)$$
$$=2\sin\alpha\cos\alpha$$
$$=2.\frac{\sqrt{13}}{4}.\frac{\sqrt{3}}{4} = \frac{\sqrt{39}}{8}$$
$$\cancel{=\frac{\sqrt{69}}{8}}$$

12(b) (i) (1 mark)
Outcomes Assessed: O3

Criteria	Mark
• Correct answer reasoning	1

Answer

$AO=OB$ (radii of circle centre O)

12(b) (ii) (2 marks)
Outcomes Assessed: O3

Criteria	Marks
• Notes that *Also* $\angle AZO=90°$ with reasons	1
• Correct answer with correct reasoning	1

Answer

If from (i) ΔOAB *is isosceles*
Also $\angle AZO=90°$ *(angles in a semi-circle are right angles at the circumference)*
$\therefore AZ=ZB$ *(a line from the apex of an isosceles triangle which meets the base at right angles, bisects the base)*

12(c) (3 marks)
Outcomes Assessed: O3

Criteria	Marks
• Obtains $\tan A+\tan B=4$ or $\tan A\tan B=9$	1
• Uses $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	1
• Correct answer	1

Answer

$\tan A+\tan B=-\frac{b}{a}$
$\tan A+\tan B=4$
$\tan A\tan B=\frac{c}{a}$
$\tan A\tan B=9$
$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$=\frac{4}{1-9}$$
$$=-\frac{1}{2}$$
$$\angle(A+B)=153°26',\ 333°26'$$
$$=153°,\ 333°$$

12(d) (i) (2 marks)
Outcomes Assessed: O4

Criteria	Marks
• Makes a positive attempt to solve obtain the remainder by long division	1
• Correct answer	1

Answer
$P(x)=x^4+3x^3+6x^2+ax+b$
By long division
$P(x)=(x^2+2x+1)(x^2+x+3)+[(a-7)x+(b-3)]$
$\therefore R(x)=(a-7)x+(b-3)$

12(d) (ii) (1 mark)
Outcomes Assessed: O4

Criteria	Mark
• Correct answer	1

Answer
$3x+2=(a-7)x+(b-3)$
$\therefore a=10$ and $b=5$

12(e) (i) (2 marks)
Outcomes Assessed: O3

Criteria	Marks
• Uses the double angle correctly at least once	1
• Correct answer with correct working	1

Answer
$\sin A\cos A\cos 2A=\frac{1}{4}\sin 4A$

$$LHS=\frac{1}{2}[2\sin A\cos A\times\cos 2A]$$
$$=\frac{1}{2}[\sin 2A\times\cos 2A]$$
$$=\frac{1}{2}\times\frac{1}{2}[2\sin 2A\times\cos 2A]$$
$$=\frac{1}{4}\sin 4A$$
$$=RHS$$

12(e) (ii) (2 marks)
Outcomes Assessed: O3

Criteria	Marks
• Achieves at least 2 of the correct answers	1
• Correct answer	1

Answer
$\frac{1}{4}\sin 4A=0$
$\sin 4A=0$
$4A=0,\pi,2\pi,3\pi,4\pi.....$
$A=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},.....$

Hence $A=0,\frac{\pi}{4},\frac{\pi}{2} \quad ;0\le A\le\frac{\pi}{2}$

Question 13 (15 marks)
13(a) (3 marks)
Outcomes Assessed: O2

Criteria	Marks
• ~~Provides clear~~ steps similar to the first three steps below	~~1~~
• Makes a substitution in step 3 ~~(from step 2)~~	1
• Provides a clear working ~~to obtain the required result and provides a conclusion~~	1

Answer

$5^n \geq 1+4n$

Step 1&3
Step 2 – Using assumption for n=k
– All correct

Step 1: Prove the expression is true for n=1
$5\ge 5$ *(true)* for n=1
LHS = $5^1=5$
RHS = $1+4\times 1=5$

Assume the expression is true for n=k (where k is even)
i.e. $5^k\ge 1+4k$ (where k is a positive integer)

Step 2 : Prove the expression is true for n=k+1
i.e. $5^{k+1}\ge 1+4(k+1)$
$5.5^k\ge 4k+5$

$5^{k+1}-4k-5\ge 0$
Now $LHS = 5.5^k-4k-5$
$\ge 5(1+4k)-4k-5$ (from assumption)
$=16k\ge 0 \qquad (k\ge 1)$

Hence if the expression is true when n=k, it is true when n=k+1
Step 3 If the expression is true for n=1, $\therefore$ it is true when n=2
If true for n=2, $\therefore$ it is true when n=3
Therefore the expression is true for all n, $n\ge 1$.

3

13(b) (i) (2 marks)
Outcomes Assessed: O4

Criteria	Marks
• Show gradient of tangent at $P = p$	1
• Achieves $y = px - ap^2$ with sufficient working	1

Answer $x^2 = 4ay$

$\frac{dy}{dx} = \frac{x}{2a} = \frac{2ap}{2a} = p$

Gradient of tangent at $P = \frac{2ap}{2a} = p$

Equation of tangent at P

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

2

13(b) (ii) (1 mark)
Outcomes Assessed: O4

Criteria	Marks
• Correct working to achieve Z	1

Answer

Tangent: $y = px - ap^2$

For Z: substitute $y=0$

$px - ap^2 = 0$

$x = ap$

$Z = (ap, 0)$

1

13(b) (ii) (2 marks)
Outcomes Assessed: O4

Criteria	Marks
• Obtains $\text{midpoint}_{ZQ} = \left(\frac{-ap}{2}, \frac{ap^2}{2}\right)$	1
• Correct answer for locus	1

Answer

$Q = (-2ap, ap^2)$ ~~symmetry of~~ parabola

$\therefore \text{midpoint}_{ZQ} = \left(\frac{-ap}{2}, \frac{ap^2}{2}\right)$ $\left(\frac{-2ap+ap}{2}, \frac{ap^2+0}{2}\right)$

hence $x = \frac{-ap}{2}$ — 1

$p = \frac{-2x}{a}$

Sub into 2,

$y = \frac{ap^2}{2}$ — 2

$= \frac{a}{2}\left(\frac{-2x}{a}\right)^2$

$y = \frac{2x^2}{a}$

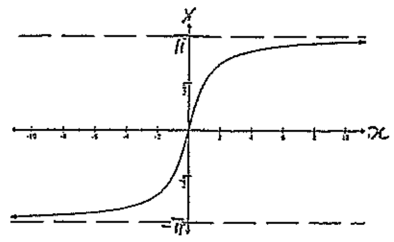
$\therefore 2x^2 = ay$ is the locus of midpoints of QZ.

2

13(c) (i) (1 mark)
Outcomes Assessed: O4

Criteria	Mark
• Correct ~~diagram~~ graph with asymptotes labeled.	1

Answer



1

13(c) (ii) (1 mark)
Outcomes Assessed: O4

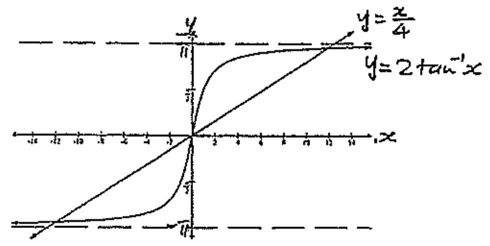
Criteria	Mark
• Correct answer	1

Answer

$2\tan^{-1}(x) - \frac{x}{4} = 0$

$2\tan^{-1}(x) = \frac{x}{4}$

$\therefore$ Graph $y = 2\tan^{-1}(x)$ and $y = \frac{x}{4}$ to show that there is only one point of intersection $(12, 3)$ for $x > 0$



1

13(c) (iii) (2 marks)
Outcomes Assessed: O4

Criteria	Marks
• ~~Uses the correct formulae and makes a good attempt at achieving answer~~	~~1~~
• ~~Correct answer~~	~~1~~

Answer

$P(x) = 2\tan^{-1}(x) - \frac{x}{4}$

$P'(x) = \frac{2}{1+x^2} - \frac{1}{4}$

$x_2 = x_1 - \frac{P(x_1)}{P'(x_1)}$

$= 10 - \frac{P(10)}{P'(10)}$

$= 10 + \frac{0.4423}{0.230}$

$= 11.92$ *(2 decimal places)*

2

13(d) (3 marks)
Outcomes Assessed: O5

Criteria	Marks
• Differentiates correctly	~~1~~
• ~~Equates various rates correctly~~	1
• ~~Achieves correct answer~~	~~1~~

Answer

$x^2 + y^2 = 25$

$y = \sqrt{25 - x^2}$ $(y > 0)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$

Now $\frac{dx}{dt} = 1$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}} \times 1$

Now when $y = 4$, $x = 3$

$\frac{dy}{dt} = \frac{-3}{\sqrt{25 - 3^2}} \times 1$

$= -\frac{3}{4}$ *(i.e. $\frac{3}{4}$ metres per second down the wall)*

3

Note: A quicker method using the implicit function rule can be used by Ext 2 students.

Question 14 (15 marks)

14(a) (i) (2 marks)
Outcomes Assessed: O5

Criteria	Marks
• ~~Achieves the acceleration formulae in any form~~	1
• ~~Correct working~~	~~1~~

Answer

$x = 2 + \sin^2 t$ — 1

$\dot{x} = v = 2\sin t \cos t$

$\ddot{x} = \cos t(2\cos t) + 2\sin t(-\sin t)$

$= 2[\cos^2 t - \sin^2 t]$

$= 2[1 - \sin^2 t - \sin^2 t]$

$= 2[1 - 2\sin^2 t]$

$= 2[1 - 2(x-2)]$ using 1

$= 2[1 - 2x + 4]$

$= 10 - 4x$

$= -4\left(x - \frac{5}{2}\right)$

Since in the form of $\ddot{x} = -n^2 X$, therefore SHM.
where $X = (x - \frac{5}{2})$ and $n = 2$

Alternative solution

$\dot{x} = \sin 2t$

$\ddot{x} = 2\cos 2t$

$= 2(1 - 2\sin^2 t)$

$= 2[1 - 2(x-2)]$ using 1

$= -4(x - \frac{5}{2})$

2

14(a) (ii) (1 mark)
Outcomes Assessed: O5

Criteria	Mark
• Correct answer	1

Answer

$\ddot{x} = 0$

$10 - 4x = 0$

$x = \frac{5}{2}$

1

14(a) (iii) (2 marks)
Outcomes Assessed: O5

Criteria	Marks
• Achieves $t=0,\frac{\pi}{2},\pi,\frac{3\pi}{2}.....$	1
• Correct answer with sufficient working	1

Answer

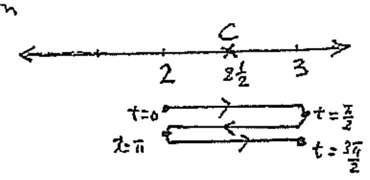
$v=0$ ← Particle changes direction

$v=2\sin t\cos t$

$2\sin t\cos t=0$

$\therefore \sin t=0 \qquad \cos t=0$

$t=0,\pi,2\pi,3\pi... \qquad t=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$



$t=0,\frac{\pi}{2},\pi,\frac{3\pi}{2}.....$ ✓

when $t=0 \quad x=2$

$t=\frac{\pi}{2} \quad x=3$

$t=\pi \quad x=2$

$t=\frac{3\pi}{2} \quad x=3$ ✓

$\therefore$ total distance $=3cm$

2

14(b) (3 marks)
Outcomes Assessed: O3

Criteria	Mark
• Find AC	1
• Find $\angle ABC$	1
• Correct answer to area	1

Answer

$\Delta ABD, \quad AB^2 = 2.4^2 + 6.4^2$
$\quad AB \approx 6.835m$

$\Delta CBD, \quad BC^2 = 2.4^2 + 5.2^2$
$\quad BC \approx 5.727m$

$\Delta ACD, \quad AC^2 = 6.4^2 + 5.2^2 - 2\times 6.4\times 5.2\times \cos 125°$ ✓
$\quad AC \approx 10.304m$

$\Delta ABC, \quad \cos\angle ABC = \frac{AB^2+BC^2-AC^2}{2(AB)(BC)}$
$\quad \approx \frac{46.72+32.8-106.17}{2(6.835)(5.727)}$

$\angle ABC \approx 109.9°$ ✓

Area $\Delta ABC = \frac{1}{2}\times 6.835\times 5.727\times \sin 109.9°$
$= 18.4\,m^2\ (1dp)$ ✓

3

14(c) (i) (2 marks)
Outcomes Assessed: O5

Criteria	Marks
• Show $\dot{x}$ and $\dot{y}$ with sufficient working	1
• Correct x and y with working shown	1

Answer

$\tan\alpha=\frac{5}{12}, \therefore \sin\alpha=\frac{5}{13}$ and $\cos\alpha=\frac{12}{13}$

$\ddot{x}=0 \qquad \ddot{y}=-10$

$\dot{x}=v\cos\alpha \qquad \dot{y}=-10t+v\sin\alpha$

$\dot{x}=130\times\frac{12}{13} \qquad \dot{y}=-10t+130\times\frac{5}{13}$

$\dot{x}=120 \qquad \dot{y}=-10t+50$

$x=120t \qquad y=-5t^2+50t+15$

$\ddot{x}=0$
$\dot{x}=C_1$
when $t=0$, $\dot{x}=v\cos\alpha$
$\therefore C_1=v\cos\alpha$
$\dot{x}=v\cos\alpha$
$=130\times\frac{12}{13}$
$=120$

$x=120t+C_3$
when $t=0$, $x=0$
$\therefore C_3=0$
$x=120t$

$\ddot{y}=-10$
$\dot{y}=-10t+C_2$
when $t=0$, $\dot{y}=v\sin\alpha$
$\therefore C_2=v\sin\alpha$
$\dot{y}=-10t+v\sin\alpha$
$=-10t+130\times\frac{5}{13}$
$=-10t+50$

$y=-\frac{10t^2}{2}+50t$
when $t=0$, $y=15$
$\therefore 15=C_4$
$y=-5t^2+50t+15$

14(c) (ii) (2 marks)
Outcomes Assessed: O5

Criteria	Marks
• Uses $v^2=\left(\dot{x}\right)^2+\left(\dot{y}\right)^2$ and works towards answer	1
• Correct answer	1

Answer

$$v^2=\left(\dot{x}\right)^2+\left(\dot{y}\right)^2$$

$\left(60\sqrt{5}\right)^2=(120)^2+(-10t+50)^2$ ✓

$18000=14400+100t^2-1000t+2500$

$100t^2-1000t-1100=0$

$t^2-10t-11=0$

$(t-11)(t+1)=0$

$t=11,-1$

$t=11\ (t>0)$

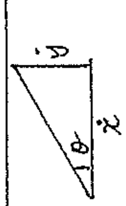
$x=120(11)$
$=1320m$ ✓

2

14(c) (iii) (3 marks)
Outcomes Assessed: O5

Criteria	Marks
• Uses $\tan\left(20°\right)\le\left\lvert\frac{\dot{y}}{\dot{x}}\right\rvert\le\tan\left(30°\right)$ or similar	1
• Flight path is in a downward direction $\therefore$ negative	1
• Correct answer	1

Answer



$\tan\theta=\frac{\dot{y}}{\dot{x}}$

$20° < \theta < 30°$

$\tan\left(20°\right)\le\left\lvert\frac{\dot{y}}{\dot{x}}\right\rvert<\tan\left(30°\right) \rightarrow \tan 20° < \left\lvert\frac{-10t+50}{120}\right\rvert < \tan 30°$ ✓

$0.364<\left\lvert\frac{-10t+50}{120}\right\rvert<0.577$

$43.68<\left\lvert-10t+50\right\rvert<69.28$

As the flight path is in a downward direction $\dot{y}<0$. $\Rightarrow 43.68 < 10t-50 < 69.28$

Hence

$50-10t<-43.68 \qquad 50-10t>-69.28$

$9.868\le t \qquad 11.928\ge t$ ✓

$9.868<t<11.928$

$9.87<t<11.93$

3

END OF PAPER